A Brief Intro to Time Complexity

Announcements

- **EX05: Dictionary Utility Functions** due Monday (March 9) at 11:59pm
- Quiz 02 in one week (March 13)!
  - Practice problems will be posted to the site shortly (by tomorrow at the latest)
  - Hybrid Review Session next Wednesday (date and time TBA soon!)
  - If you take your quizzes with the UCO, reminder to check that you have scheduled your quiz
  - Have a university-approved absence? Email me to schedule an alternate time to take it!

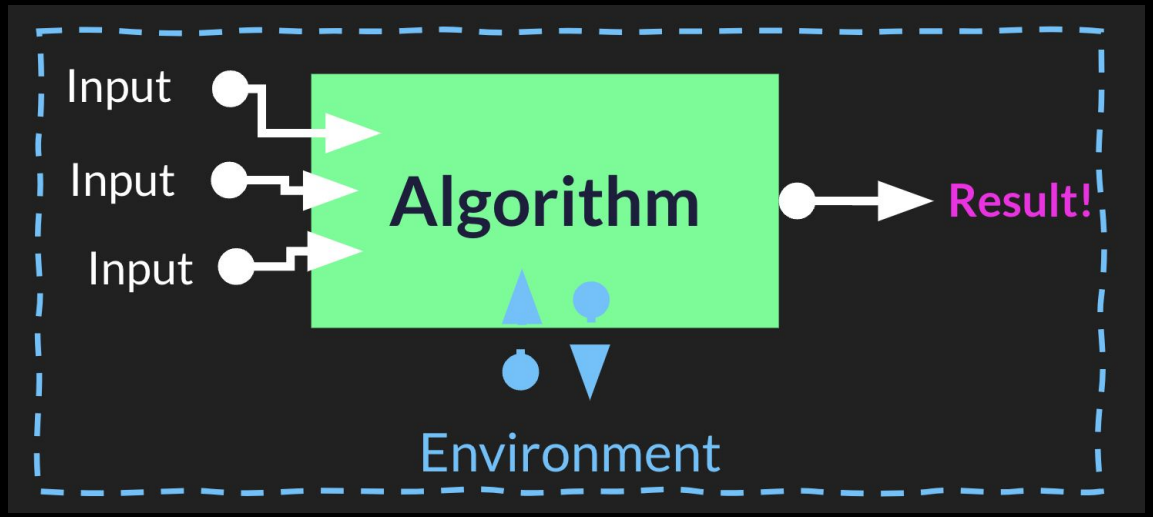
Review: Algorithms

Input is data given to an algorithm

An **algorithm** is a series of steps

An algorithm **returns** some **result**

An algorithm *may* be influenced by its **environment** and it *may* produce side-effects which influence its environment.



Warm-up:

With a partner, discuss which of the following factors you think are *most important* to consider when selecting or implementing an algorithm:

- Simplicity
- Ease of implementation
- Speed (how long does the code take to run?)
- Efficient use of memory
- Precision in answer

Is one factor *always* more important than the others, or would it vary by algorithm?

With a neighbor, try diagramming:

```
1   def intersection(a: list[str], b: list[str]) -> list[str]:
2       result: list[str] = []
3
4       idx_a: int = 0
5       while idx_a < len(a):
6           idx_b: int = 0
7           found: bool = False
8           while not found and idx_b < len(b):
9               if a[idx_a] == b[idx_b]:
10                  found = True
11                  result.append(a[idx_a])
12              idx_b += 1
13          idx_a += 1
14
15      return result
16
17
18  foo: list[str] = ["a", "b"]
19  bar: list[str] = ["c", "b"]
20  print(intersection(foo, bar))
```

… and after diagramming:

Assume our unit of "operation" is the number of times the block of lines #9-12 are evaluated.

Q1. Can different values of a and b lead to a difference in the number of operations required for the intersection function evaluation to complete?

Q2. If so, provide example item values for a and b which require the fewest operations to complete? Then try for the maximal operations to complete?

Q3. Assuming the item values of a and b are random and unpredictable, about how many operations does this function take to complete?

```python
def intersection(a: list[str], b: list[str]) -> list[str]:
    result: list[str] = []

    idx_a: int = 0
    while idx_a < len(a):
        idx_b: int = 0
        found: bool = False
        while not found and idx_b < len(b):
            if a[idx_a] == b[idx_b]:
                found = True
                result.append(a[idx_a])
            idx_b += 1
        idx_a += 1

    return result


foo: list[str] = ["a", "b"]
bar: list[str] = ["c", "b"]
print(intersection(foo, bar))
```

As the lengths of `a` and `b` grow, the number of operations grows *quadratically*

```python
def intersection(a: list[str], b: list[str]) -> list[str]:
    result: list[str] = []

    idx_a: int = 0
    while idx_a < len(a):
        idx_b: int = 0
        found: bool = False
        while not found and idx_b < len(b):
            if a[idx_a] == b[idx_b]:
                found = True
                result.append(a[idx_a])
            idx_b += 1
        idx_a += 1

    return result


foo: list[str] = ["a", "b"]
bar: list[str] = ["c", "b"]
print(intersection(foo, bar))
```

- Outer while loop iterates through each element of `a`
  - *If there are N elements, we'll iterate N times*
- And within each iteration of the outer while loop…
- The inner while loop iterates through elements of `b` until either:
  - We find a value that == the current element in `a` OR,
  - We have "visited" (accessed) every element in `b`
    - *If there are M elements in `b`, we'll iterate up to M times*

Assuming `a` and `b` both have 3 elements…

1. Example of values of `a` and `b` that will cause the ***fewest*** operations to occur?
   `intersection(a=["a", "a", "a"], b=["a", "b", "c"])`
2. Example of values of `a` and `b` that will cause the ***most*** operations to occur?
   `intersection(a=["a", "b", "c"], b=["d", "e", "f"])`

If list `a` has N elements and list `b` has M elements, the "worst case scenario" is that this code will cause N*M operations to occur.

Comparing lists and sets

```python
def intersection(a: list[str], b: list[str]) -> list[str]:
    result: list[str] = []

    idx_a: int = 0
    while idx_a < len(a):
        if a[idx_a] in b:
            result.append(a[idx_a])
        idx_a += 1

    return result
```

```python
def intersection(a: list[str], b: set[str]) -> set[str]:
    result: set[str] = set()

    idx_a: int = 0
    while idx_a < len(a):
        if a[idx_a] in b:
            result.add(a[idx_a])
        idx_a += 1

    return result
```

Suppose **`a`** and **`b`** each had 1,000,000 elements. The worst case difference here is approximately 1,000,000 operations, versus 1,000,000**2 or 1,000,000,000,000 operations.

If your device can perform 100,000,000 operations per second, then...

A call to **`a`** will complete in 2.78 hours and **`b`** will complete in 1/100th of a second.

Recall: comparing lists and sets

```python
def intersection(a: list[str], b: list[str]) -> list[str]:
    result: list[str] = []

    idx_a: int = 0
    while idx_a < len(a):
        if a[idx_a] in b:
            result.append(a[idx_a])
        idx_a += 1

    return result
```

```python
def intersection(a: list[str], b: set[str]) -> set[str]:
    result: set[str] = set()

    idx_a: int = 0
    while idx_a < len(a):
        if a[idx_a] in b:
            result.add(a[idx_a])
        idx_a += 1

    return result
```

Suppose **`a`** and **`b`** each had 1,000,000 elements. The worst case difference here is approximately 1,000,000 operations, versus 1,000,000**2 or 1 trillion (1,000,000,000,000) operations.

If your device can perform 100,000,000 operations per second, then...

A call to **`a`** will complete in 2.78 hours and **`b`** will complete in 1/100th of a second.

Running time: how long does an algorithm take to run?

- Empirical analysis: write the code and test how long it takes to run!
  - Weaknesses:
    - You have to write the code for the whole algorithm and run it to see how long it will take
    - Different computers with different specs will have different runtimes
- Rather than using empirical analysis, computer scientists commonly consider the **number of operations (steps)** an algorithm requires
  - 1 operation == 1 step

Runtime analysis: Best, average, and worst case

Best case (lower bound):

- Minimum number of operations (running time) required for the algorithm to execute

Average case:

- Average running time among several different inputs

Worst case (upper bound) ✨:

- The maximum running time given an input
  - How does the number of operations grow as an input grows?
- Important to understand how our algorithm will perform in the *worst* case
  - Prepare for the worst case. If an input ends up requiring fewer operations, great!!

Runtime analysis: order of growth

Runtime analysis: order of growth